SIMULATION OF DEFORMATION OF AN ELASTIC CAPSULE IN FLUID FLOW USING IMMERSED BOUNDARY METHOD

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ABSTRACT

The deformation and transport of elastic capsule, the simplest model of red blood cell, in fluid flow inside a channel is of significant research interest because of the various applications in the areas of biomicrofluidics and biomedical engineering. Being a fluid-structure interaction problem, developing a computational model for the study of capsule interaction with fluid flow is quite challenging. The present paper discuss a two-dimensional computational model developed using immersed boundary method to investigate the transport and deformation dynamics of an elastic capsule in a channel flow under different Reynolds numbers and initial capsule locations. The capsule is modeled using immersed boundary points forming a network of elastic links which can undergo tension/compression and bending. The fluid flow is modeled using continuity and Navier-Stokes equations with finite volume method based discretization on a staggered grid system. We compute Taylor deformation parameter to assess the significance of Reynolds number and initial location of the capsule for the deformation behavior. Simulations are performed for two different Reynolds numbers and three different initial locations. The final shape and position of the capsule are well captured using the developed model.

Keywords: Elastic capsule; Finite volume method; Immersed boundary method, Reynolds number, Taylor deformation parameter

INTRODUCTION

Elastic capsule can be used as a simple model to study red blood cell deformation and transport in the field of biomedical engineering. The problem of elastic capsule deformation and transport under pressure driven flow in a channel involves fluid-structure interaction. Hence, it has lots of research potential both from the fundamental research and industrial applications point of view. Numerous experimental, theoretical and computational studies have been done in the area of capsule transport and deformation in various conditions of channel flow. For artificial capsules flowing through a microfluidic channel the classical slug shape has been widely noticed in experimental studies [1, 2]. The axisymmetric parachute-like shapes for the red blood cell is observed under channel flow due to the influence of fluid dynamic forces, elasticity of the capsule membrane and boundary configuration [3]. Barthes-Biesel employed asymptotic expansions to study the deformation behaviour of a spherical capsule in simple shear flow [4]. Secomb et al. [5] through lubrication theory studied the flow in a narrow cylindrical channel with a capsule undergoing steady axisymmetric deformation. Ramanujan and Pozrikidis [6] developed three-dimensional boundary element method based model and analysed the effects of fluid viscosity on capsule deformation under shear flow. Using boundary element method Diaz et al. [7] observed the shape changes and transport dynamics of a capsule. Secomb et al. [8] suggested a new finite element based computational model for a red blood cell by adding a set of interconnected viscoelastic elements to model the membrane. Sui et al. [9] investigated the effect of capsule membrane bending stiffness in the dynamics of capsule under shear flow. The effects of elasticity, initial capsule shape, and initial capsule position for a capsule motion in a channel were analysed by Ma *et al.* [10]. Shin et al. [11] studied the inertial migration of an elastic capsule in a channel flow using feedback forcing based immersed boundary method. Song et al. [12] discussed the transient behaviour of a circular capsule in three viscous shear flows for different Reynolds number and capillary number. Motivated by previous works, we present a simple two-dimensional numerical model to simulate the dynamics of an elastic capsule in a channel flow using immersed boundary method. The model is simple because we assume the capsule as neutrally buoyant and the density ratio of fluid inside and outside capsule is equal to 1.0. To the best of our knowledge, an immersed boundary finite volume method based computational model addressing the present problem of elastic capsule deformation has not been reported till now. We investigate the dynamic behaviour of capsule under different Reynolds number and initial locations by capturing the shape and final position of the capsule and computing Taylor deformation parameter.

MATHEMATICAL MODELING AND NUMERICAL PROCEDURE

Figure 1 shows the physical problem of elastic capsule immersed in a fluid flow in a channel. We use circle as the initial shape of the capsule and the initial location is at (x_o, y_o) in a channel with dimensions L x H, where L is the length of the channel and His height of the channel. The present work use IB method proposed by Peskin [13].We use Navier-Stokes and continuity equations in its dimensionless form with usual notations which are as follows

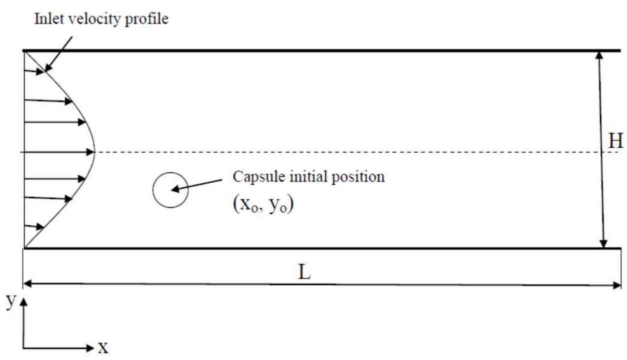


FIGURE 1: SCHEMATIC REPRESENTATION OF ELASTIC CAPSULE IMMERSED IN A FLUID FLOW IN A CHANNEL

The Eulerian force density $\mathbf{f}$ in eqn. (1) is given by

$$\mathbf{f}(\mathbf{x},t) = \int \mathbf{F}(s,t)\ \delta(\mathbf{x} - \mathbf{X}(s,t))\, ds \qquad (3)$$

where $\mathbf{F}$ is the Lagrangian force density acting on the capsule and $\delta(\mathbf{x} - \mathbf{X}(s,t))$ is the two-dimensional Dirac delta function. Refer to our previous works to see the detailed step-by-step procedure to compute the Lagrangian force density $\mathbf{F}$ [14,15].

The governing equations (1) and (2) are solved using fractional step method with the help of finite volume discretization on a staggered Cartesian grid system [Refer to [14,15] for more details]. The fluid velocity obtained after solving equations (1) and (2) are then applied to determine Lagrangian velocity with the help of Dirac delta function as shown below

$$\mathbf{U}(s,t) = \int \mathbf{u}(\mathbf{x},t)\ \delta(\mathbf{x} - \mathbf{X}(s,t))\, d\mathbf{x} \qquad (4)$$

The obtained Lagrangian velocity is used to find the new position $\mathbf{X}_k^{n+1}$ of an IB point say $\mathbf{X}_k^n$ in the following form

$$\mathbf{X}_k^{n+1} = \mathbf{X}_k^n + \Delta t\ \mathbf{U}(s,t) \qquad (5)$$

where Δt is the time step-size.

RESULTS AND DISCUSSION

A FORTRAN code is built to simulate deformation behavior of elastic capsule in a channel flow. We consider dimensionless channel size of 16 x 2. The initial diameter of capsule is taken as 0.4. A parabolic velocity profile is assumed at the inlet of the channel. Capsule deformation is measured by the Taylor deformation parameter defined as $D_{xy} = \frac{(L_A - L_B)}{(L_A + L_B)}$ where L_A and L_B are, respectively, the maximum and minimum diameters passing through the center of the elongated capsule. It is used as a relevant parameter for the quantitative assessment of capsule deformation in a fluid flow.

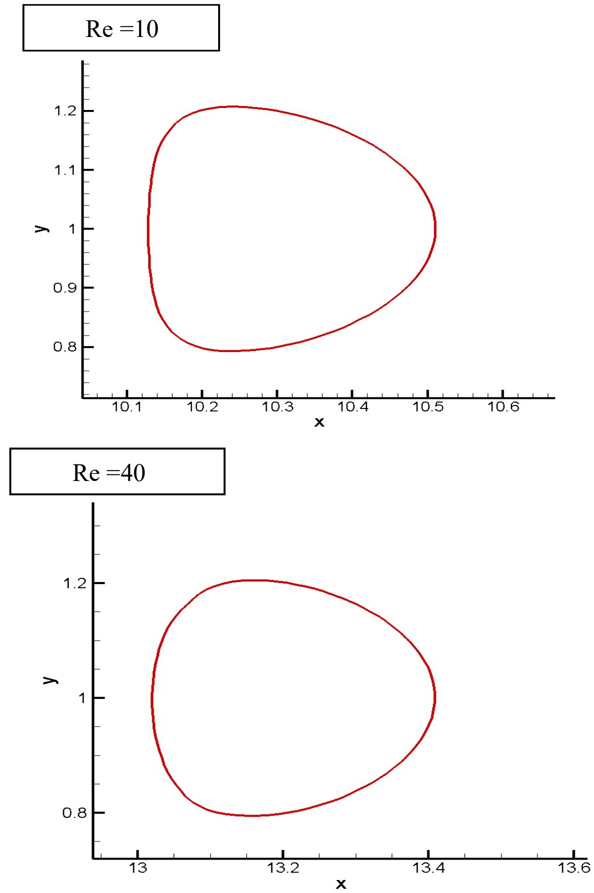


FIGURE 2: SHAPE AND POSITION OF ELASTIC CAPSULE FOR TWO DIFFERENT REYNOLDS NUMBERS AT DIMENSIONLESS TIME T=15.0 AT INITIAL LOCATION $x_c = 1.0, y_c = 1.0$

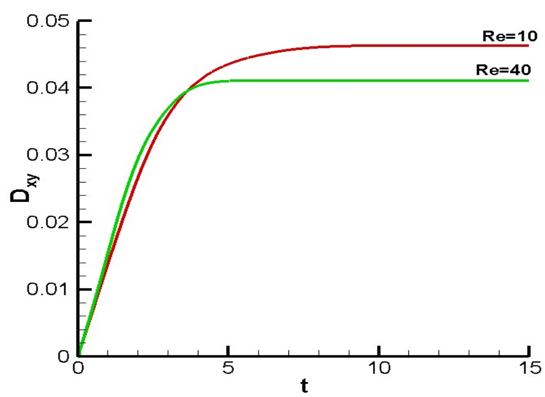
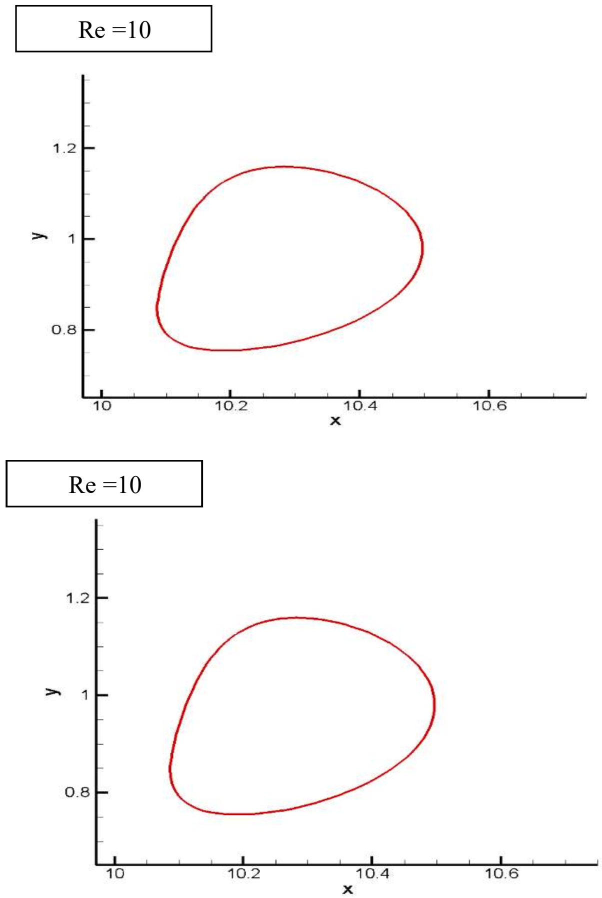


FIGURE 3: PLOT OF TAYLOR DEFORMATION PARAMETER WITH RESPECT TO DIMENSIONLESS TIME FOR TWO DIFFERENT REYNOLDS NUMBERS AT $x_c = 1.0, y_c = 1.0$.



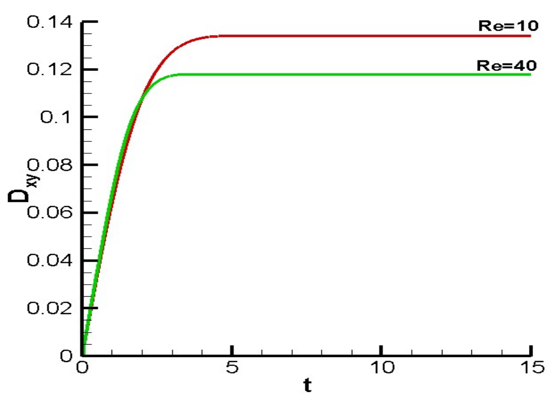


FIGURE 5: PLOT OF TAYLOR DEFORMATION PARAMETER WITH RESPECT TO DIMENSIONLESS TIME FOR TWO DIFFERENT REYNOLDS NUMBER AT $x_c = 1.0, y_c = 0.95$

Further, we carried out simulations for the cases of off center capsules for Re=10 and 40. In the first case the capsule is placed at$x_c = 1.0, y_c = 0.95$. Figure 4 shows the final shape and position of the capsule and figure 5 shows the variation of Taylor deformation parameter with simulation time. From figure 4 it can be observed that the capsule undergoes large deformation compared with the case of center line capsule for both Reynolds numbers. From figure 5, we computed the values of Taylor deformation parameter for Re=10 as 0.134 and for Re=40 as 0.118. Hence, it is further verified quantitatively that off center capsule undergoes large deformation compared to center line capsule depicted in the previous case.

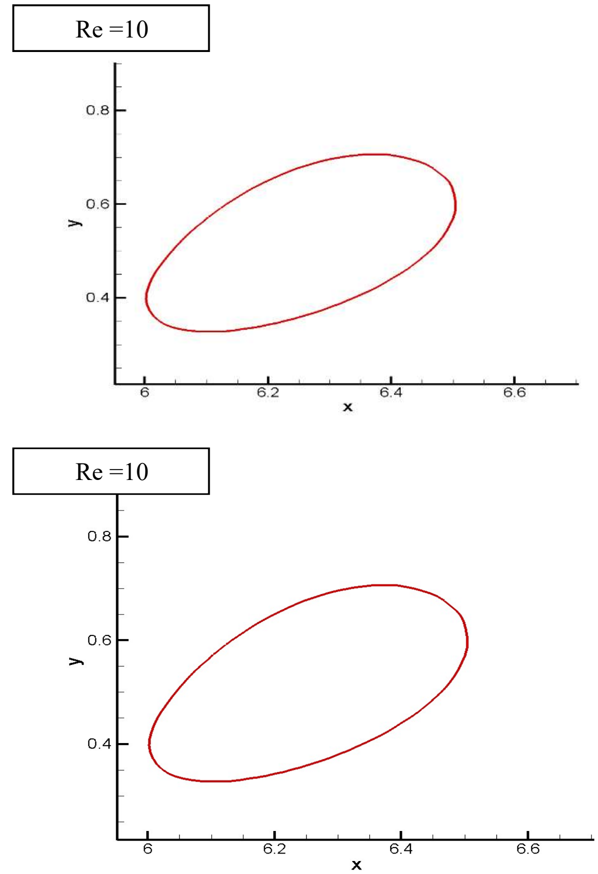


FIGURE 6: SHAPE AND POSITION OF ELASTIC CAPSULE FOR TWO DIFFERENT REYNOLDS NUMBERS AT DIMENSIONLESS TIME T=15.0 AT INITIAL LOCATION $x_c = 1.0, y_c = 0.25$

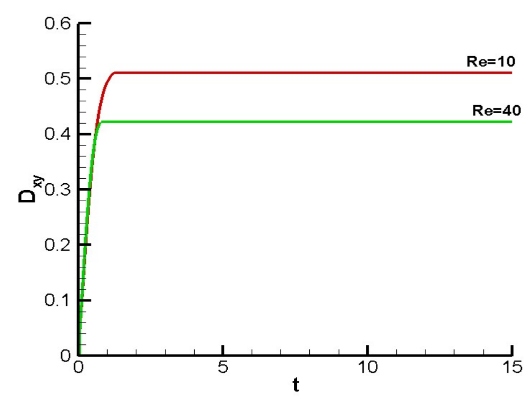


FIGURE 7: PLOT OF TAYLOR DEFORMATION PARAMETER WITH RESPECT TO DIMENSIONLESS TIME FOR TWO DIFFERENT REYNOLDS NUMBER AT $x_c = 1.0, y_c = 0.25$

Finally, we performed simulations for the case of capsule location $x_c = 1.0, y_c = 0.25$. Here, the capsule is close to the bottom of the channel wall and far away from the center. Figure 6 shows the final shape and position of the capsule under this conditions for Re=10 and 40. It can be observed that the capsule undergoes extensive deformation for both Re values compared to previous cases. Also, the capsule tends to migrate towards the center of the channel. This migration behavior is fast in the case of Re=40 compared to Re =10. Figure 7 illustrates the variation of Taylor deformation parameter with time and the values are computed to be 0.510 for Re=10 and 0.422 for Re=40. A higher value of deformation parameter justifies large deformation behavior of the capsule when it is placed far away from the center of the channel.

CONCLUSION

The present work investigates the role of Reynolds number and initial capsule location for the case of elastic capsule transport and deformation in a channel flow. Accordingly, we construct a two-dimensional numerical model using immersed boundary method to carry out the simulations and the dynamic behavior is assessed with the help of Taylor deformation parameter. Through our developed model, we found that the transport and deformation dynamics of elastic capsule in a channel flow strongly depends on Reynolds number and initial location of the capsule. Center line capsule deform to a greater extend for low values of Reynolds number. In the case of off center capsule, placing the capsule far away from the channel walls will result in the migration of capsule towards the center of the channel and also capsule undergoes severe deformation compared to the case of center line capsule. We believe that the developed immersed boundary model can be easily employed to investigate the dynamics of Red Blood Cell (RBC) under low Reynolds number flow conditions by modeling RBC as an elastic capsule with exact physiological data.

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